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The Role of the Backside Parameter in Height Control

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Height control in continuous gust fields is formulated as a stochastic regulator problem and the relation between mean square (ms) height error and the backside parameter is examined. The following results are shown. First, when the throttle loop is open, elevator alone ceases to be a reasonable control strategy when the backside parameter $1/T_\gamma$ is negative. The minimum achievable ms height error is essentially only a function of $1/T_\gamma$; ms error is zero when $1/T_\gamma > 0$, while it is inversely proportional to $|1/T_\gamma|$ when $1/T_\gamma < 0$. For the landing approach, this holds true approximately, regardless of airplane type. Second, when the throttle loop is closed, it is generally difficult to tightly control velocity and height simultaneously in conventional airplanes. This is mainly because the thrust change available in control is quite limited, particularly in deceleration. To realize tight control in height, the accuracy in velocity control must be relaxed and the backside parameter has to be properly large.

Introduction

THE backside parameter $1/T_\gamma$ has been known to be correlated with flight difficulty in the low-speed region, and several explanations have been given concerning the physical meaning of this parameter (e.g., Refs. 1-4). However, these explanations are usually qualitative and the effect of this parameter does not seem to be well understood, particularly when the throttle and the elevator are used simultaneously. This paper tries to explain the role of this parameter in airplane height control in a quantitative manner, employing the basic assumption that the pilot is a stochastic optimal regulator for disturbances due to gusts.

This paper consists mainly of two parts. After summarizing basic equations used in the analyses, the dynamics with the throttle loop open are discussed first. It is shown that the maximum achievable accuracy in terms of minimum mean square height error is essentially only a function of the backside parameter $1/T_\gamma$, regardless of airplane type, for the landing approach flight phase. The main result of this part is given as Eqs. (31) or (32), which is the solid line in Fig. 1.

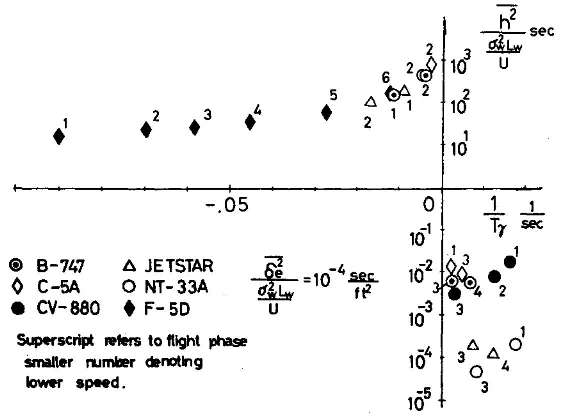

Fig. 1 Comparison between regulator and filter analysis.

Then, the effect of throttle loop closure is discussed. It should be borne in mind that there is a distinct difference between the flight-path angle control and the height control when only the elevator is utilized. In the flight-path angle control, mean square path-angle error increases with $|1/T_\gamma|$ when $1/T_\gamma$ is negative, as was shown in Ref. 5. On the contrary, height control by elevator alone is most critical when $1/T_\gamma$ is negative and close to zero, as shown in Fig. 1. In this case, throttle control has to be included. It should also be borne in mind that the throttle has to control airplane velocity as well as height, and this has to be reflected in the analysis. The main purpose of this latter part is to see how the relation given by Eq. (31) or (32) will be changed by the introduction of the throttle control, particularly when the mean square velocity error and the mean square throttle available are specified.

Basic Equations

Using standard notation, the following equations of motion are used:

$$\begin{bmatrix} (1-X_{\dot{u}})S-X_u, & -X_{\dot{w}}S-X_w, & (-X_q+W_0)S+g\cos\theta_0 \\ -Z_{\dot{u}}S-Z_u, & (1-Z_{\dot{w}})S-Z_w, & (-Z_q-U_0)S+g\sin\theta_0 \\ -M_{\dot{u}}S-M_u, & -(M_{\dot{w}}S+M_w), & S^2-M_qS \end{bmatrix} \begin{bmatrix} u \\ w \\ \theta \end{bmatrix}$$

$$= \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \cdot \delta_e + \begin{bmatrix} 1 \\ Z_{\delta_T}/X_{\delta_T} \\ M_{\delta_T}/X_{\delta_T} \end{bmatrix} \cdot T - \begin{bmatrix} X_u, & X_w \\ Z_u, & Z_w \\ M_u, & M_w \end{bmatrix} \begin{bmatrix} u_g \\ \\ w_g \end{bmatrix} \tag{1}$$

$$\dot{\theta} = q \tag{2}$$

$$\dot{h} = -w\cos\theta_0 + u\sin\theta_0 + (U_0\cos\theta_0 + \overline{W}_0\sin\theta_0)\theta \tag{3}$$

The equations are referred to body axes, and these equations are identical to those shown in Ref. 6 (page C-1), except for the terms concerned with thrust T and gusts u_g and w_g. T is described as $X_{\delta_T} \cdot \delta_T$ in standard texts. In this paper, thrust T is approximated by the following first-order system, with time constant T_L.

$$T_L\dot{T} + T = X_{\delta_T}\delta_T \tag{4}$$

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Horizontal and vertical gust velocities, u_g and w_g, are approximately simulated by the following first-order systems:

$$T_u \dot{u}_g + u_g = \eta_u \tag{5}$$

$$T_\omega \dot{w}_g + w_g = \eta_w \tag{6}$$

where T_u and T_w are the time constants of the respective systems, and n_u and n_w are Gaussian white noise processes whose intensities are I_{u_g} and I_{w_g}, respectively. u_g and w_g are assumed to be uncorrelated.

The time constants and intensities of u_g and w_g are determined by the low-frequency power spectrum and the mean square gust velocity (noted as σ_u^2 and σ_w^2) of the gust model to be simulated. When the Dryden model is matched, one obtains

$$T_u = L_u/U \qquad I_{u_g} = 2\sigma_u^2 L_u/U$$
$$T_w = L_w/(2U) \qquad I_{w_g} = \sigma_w^2 L_w/U \tag{7}$$

where U denotes trim speed $(\sqrt{U_0^2+W_0^2})$, while L_u and L_w denote the scale lengths of the horizontal and vertical gust, respectively.

Controls are defined as those of an optimal regulator. An appropriate stochastic optimal regulator problem is now formulated. For definitions and the detailed solution of this problem, the reader is referred to Ref. 8 (pp. 253-265).

The system equation can be constructed for Eqs. (1-6) in the following form:

$$\dot{x} = A\cdot x + B\cdot u + C\cdot v \tag{8}$$

where x, u, and v denote state vector, control vector, and Gaussian white noise vector, respectively. The criterion functional J to be minimized is

$$J = \overline{x^T R_1 x} + \overline{u^T R_2 u} \tag{9}$$

where $\overline{(\ )}$ denotes the expected value.

The steady-state solution of this problem is given as

$$u(t) = -R_2^{-1}B^T P\cdot x(t) \equiv -F\cdot x(t) \tag{10}$$

where P is the positive definite solution of

$$0 = R_1 - PBR_2^{-1}B^T P + A^T P + PA \tag{11}$$

If the steady-state variance matrices of x and u are denoted by Q and S, that is,

$$\overline{x\cdot x^T} = Q \qquad \overline{u\cdot u^T} = S \tag{12}$$

then Q and S are computed from

$$0 = (A-BF)Q + Q(A-BF)^T + CVC^T \tag{13}$$

$$S = FQF^T \tag{14}$$

from which the mean square values of states and controls can be obtained.

Height Control with Throttle Loop Open

Flight through vertical gust is considered and with elevator as the only available control. The state equation (8) is composed of Eqs. (1-3) and (6), where $T = u_g = 0$ is assumed in Eq. (1). State vector x, control u, and intensity V of the disturbance v are as follows:

$$x = [u, w, q, \theta, h, w_g]^T$$
$$u = \delta_e \qquad V = I_{w_g} \tag{15}$$

The criterion functional J to be minimized is taken as

$$J = \overline{h^2} + \rho\overline{\delta_e^2} \tag{16}$$

where ρ is the relative weight for control magnitude.

The solution gives, among other things, the mean square (ms) of height error $\overline{h^2}$ and control $\overline{\delta_e^2}$ for given values of ρ and gust intensity $\sigma_w^2 L_w/U$. Generally, the decrease in ρ reduces $\overline{h^2}/(\sigma_w^2 L_w/U)$, the ms height error normalized to gust intensity, and increases $\overline{\delta_e^2}/(\sigma_w^2 L_w/U)$, the ms control normalized to gust intensity. Therefore, in studying the parametric effect of the backside parameter $1/T_\gamma$ upon normalized ms height error $\overline{h^2}/(\sigma_w^2 L_w/U)$, either ρ or normalized ms control $\overline{\delta_e^2}/(\sigma_w^2 L_w/U)$ must be specified. As a rough measure, the following is proposed:

$$\overline{\delta_e^2}/(\sigma_w^2 L_w/U) = 10^{-4}\text{s/ft}^2 \tag{17}$$

This gives

$$\sqrt{\overline{\delta_e^2}} \cong 2.5 \text{ deg} \tag{18}$$

for

$$\sigma_w = 2\text{ fps} \qquad L_w = 1000\text{ ft} \qquad U = 200\text{ fps} \tag{19}$$

which is a typical gusty condition near landing approach.

Numerical calculations are conducted for several airplanes for flight conditions near landing approach and the values of $\overline{h^2}/(\sigma_w^2 L_w/U)$ are determined when $\overline{\delta_e^2}/(\sigma_w^2 L_w/U) = 10^{-4}$ s/ft^2 is reached. They are plotted against $1/T_\gamma$ in Fig. 1 with a linear scale on the ordinate, and in Fig. 2 with a logarithmic scale on the ordinate. The stability derivatives necessary for these calculations were obtained from Refs. 6 and 7. Although not shown here, it has been ascertained that $\overline{h^2}/(\sigma_w^2/L_w U)$ obtained when Eq. (17) holds is pretty close to one obtained when no limitations are imposed on control (that is, $\rho \to 0$ in Eq. (16)).

Figures 1 and 2 show that the best achievable performance depends essentially on $1/T_\gamma$ regardless of other airplane parameter values. Furthermore, control by means of the elevator alone will be particularly difficult when $1/T_\gamma$ is negative and close to zero.

To facilitate understanding, a simplified analysis is also presented. If the pitch stabilization is tight enough, elevator control is equivalent to specifying the pitch attitude via a commanded value θ_c, and the perturbed velocity u, flight-path angle γ, and height h will be approximately given by

$$\begin{bmatrix} S-X_u, & 0, & 0 \\ Z_u/U, & S-Z_w, & 0 \\ 0, & -U, & S \end{bmatrix} \begin{bmatrix} u \\ \gamma \\ h \end{bmatrix} = \begin{bmatrix} -g \\ -Z_w \\ 0 \end{bmatrix} \cdot\theta_c + \begin{bmatrix} 0 \\ -Z_w/U \\ 0 \end{bmatrix} w_g \tag{20}$$

where the same notation is used as in Eq. (1). Vertical gust w_g is again given by Eq. (6), but is measured positive upward.

Optimal control is defined to minimize the following criterion J', which is the modified form of Eq. (16):

$$J' = \overline{h^2} + \rho'\overline{\theta_c^2} \tag{21}$$

This can be formulated as a Wiener filter problem, as shown in Fig. 3. The problem is to find the optimum filter transfer function $K(s)$ which minimizes Eq. (21), where $h = h_g - h_f$, h_g is the height response (without control) to gust (w_g), and h_f is the height response produced by the filter output (θ_c). It is assumed that w_g can be measured. The transfer functions $F(s)$ and $G(s)$ can be obtained from Eq. (20) and have the following forms:

$$F(s)=\frac{(-Z_w)}{S(S-Z_w)} \tag{22}$$

$$G(s)=\frac{U(-Z_w)(S+1/T_\gamma)}{S(S-X_u)(S-Z_w)} \tag{23}$$

where

$$\frac{1}{T_\gamma}=-X_u-\frac{g}{U}\left(\frac{Z_u}{Z_w}\right) \tag{24}$$

The solution of this problem can be found, for example, in Ref. 9. Applying Eqs. (4-28) of Ref. 9 (page 70) to Fig. 3, one obtains

$$\frac{K(s)\cdot G(s)}{P(s)}=\frac{1}{P(-s)Y(s)Z(s)}\left[\frac{P(-s)F(s)\Phi_{w_g}(s/i)}{P(s)Y(-s)Z(-s)}\right]_+ \tag{25}$$

where $P(s)$, $Y(s)$, and $Z(s)$ of this problem are

$$P(s)=\begin{cases}1 & (1/T_\gamma>0)\\ 1-(s/|1/T_\gamma|) & (1/T_\gamma<0)\end{cases} \tag{26}$$

$$Y(s)=\left\{1+\frac{\rho'}{G(-s)\cdot G(s)}\right\}^+ \quad Z(s)=\left\{\Phi_{w_g}\left(\frac{s}{i}\right)\right\}^+ \tag{27}$$

$\Phi_{w_g}(\omega)$ is the gust power spectrum given from Eqs. (6) and (7) as follows:

$$\Phi_{w_g}(\omega)=\frac{I_{w_g}}{1+T_w^2\omega^2}=\frac{(\sigma_w^2L_w/U)}{1+T_w^2\omega^2} \tag{28}$$

$\{\ \}^+$ denotes a function of spectral factoring with all poles and zeros in the left-half s plane (LHP), while $[\ ]_+$ represents the process of partial fractioning, retaining only terms with poles in the LHP.

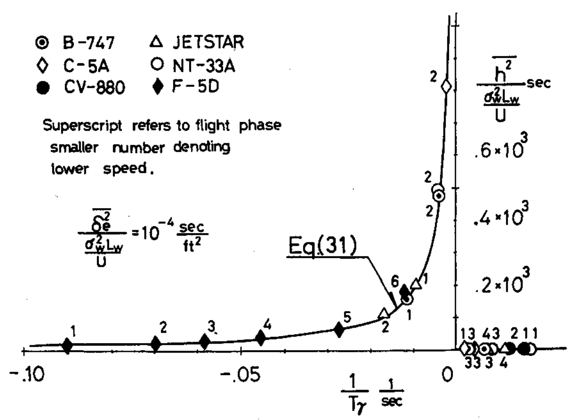

Fig. 2 Mean square height vs $1/T_\gamma$.

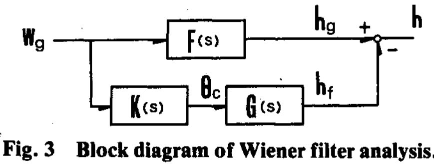

Fig. 3 Block diagram of Wiener filter analysis.

Maximum achievable accuracy can be obtained in the limit of $\rho'\to 0$ (no limitation on control magnitude). In this particular case, Eq. (25) leads to the following result:

$$K(s)=\begin{cases}\dfrac{S-X_u}{U(S+|1/T_\gamma|)} & (1/T_\gamma>0)\\[2ex] \dfrac{S-X_u}{U(S+|1/T_\gamma|)}\cdot H(s) & (1/T_\gamma<0)\end{cases} \tag{29}$$

where

$$H(s)=[-2T_wS^2-2\{1+T_w(-Z_w+|1/T_\gamma|)\}S-(-Z_w+|1/T_\gamma|)(1+T_w|1/T_\gamma|)]/[(-Z_w+|1/T_\gamma|)(1+T_w|1/T_\gamma|)] \tag{30}$$

The associated mean square height error can be calculated as follows:

$$\frac{\overline{h^2}}{\sigma_w^2L_w/U}=\begin{cases}0 & (1/T_\gamma>0)\\[2ex] 2\left(\dfrac{-Z_w}{-Z_w+|1/T_\gamma|}\right)^2\cdot\left(\dfrac{1}{1+T_w\cdot|1/T_\gamma|}\right)^2\cdot\dfrac{1}{|1/T_\gamma|} & (1/T_\gamma<0)\end{cases} \tag{31}$$

Equation (31) is plotted in Fig. 1 as a solid line, assuming $Z_w=-0.7\ s^{-1}$ and $T_w=2.5$ s. The curve fits those $\overline{h^2}/(\sigma_w^2L_w/U)$ vs $1/T_\gamma$ points obtained from regulator theory for several airplanes; this verifies the simplified analysis, as well as illustrates the role of the backside parameter expressed in Eq. (31). It should be noted that if $1/T_\gamma=0$, the system is "uncontrollable" when the problem defined in Fig. 3 is reformulated as a standard regulator problem. It should also be noted that if $1/T_\gamma<0$, the system performance degrades even if no limitations are imposed on control, as is shown in Ref. 8 (pp. 306-310).

Usually, it can be assumed that $|1/T_\gamma|\ll(-Z_w)$ and $T_w\cdot|1/T_\gamma|\ll 1$, and Eq. (31) can also be approximated as

$$\frac{\overline{h^2}}{\sigma_w^2L_w/U}\cong\begin{cases}0 & (1/T_\gamma>0)\\[1ex] \dfrac{2}{|1/T_\gamma|} & (1/T_\gamma<0)\end{cases} \tag{32}$$

Equation (32), when plotted in Fig. 2, overlays the plot of Eq. (31).

Height Control with Throttle Loop Closed

State equation consists of Eqs. (1-6), where state vector x, control u, and intensity matrix V of the disturbance v are as follows:

$$x=[u,w,q,\theta,h,u_g,w_g,T]^T$$

$$u=[\delta_e,\delta_T]^T$$

$$V=\begin{bmatrix}I_{u_g}, & 0\\ 0, & I_{w_g}\end{bmatrix} \tag{33}$$

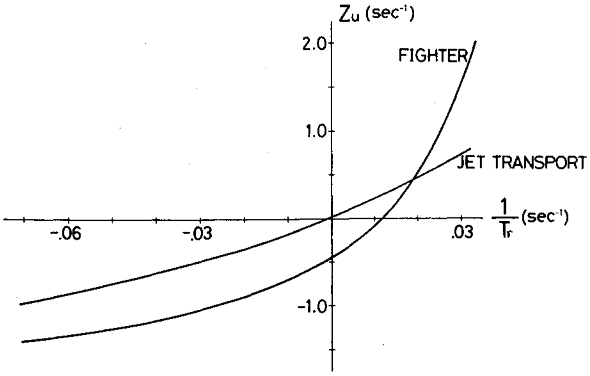

Fig. 4 Relations between $1/T_\gamma$ and Zu.

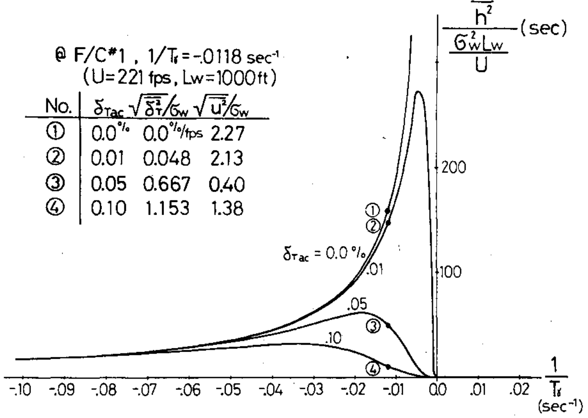

Fig. 5 Effect of throttle loop closure.

The functional J to be minimized is assumed to have the following form:

$$J=\overline{h^2}+(\overline{u^2}/u_{ac}^2)+(\overline{\delta_e^2}/\delta_{e_{ac}}^2)+(\overline{\delta_T^2}/\delta_{T_{ac}}^2) \qquad (34)$$

where terms with subscript "ac" (acceptable level) are the weights normalized to $h_{ac}=1$ ft.

Numerical calculations are undertaken primarily for a large jet transport aircraft, where the effect of throttle loop closure is discussed in some detail. Also included are less detailed numerical results for a fighter aircraft.

The stability derivatives selected for the jet transport are close to those of a B-747. Derivatives are obtained from Ref. 6 for flight condition #1, $U=221$ fps, except for Z_u and X_{δ_T}. Z_u was intentionally varied in order to simulate the effect of the variations in $1/T_\gamma$, which was obtained as a zero of the elevator to flight path transfer function. The relation between $1/T_\gamma$ and Z_u is shown in Fig. 4. It should be noted, however, that the Z_u in the right-hand side of Eq. (1) is kept constant so that the gust forcing effect is unchanged. X_{δ_T} is chosen so that a $\pm 100\%$ variation in δ_T produce $\pm 0.1\ g$ variation in thrust, taking a maximum value of δ_T as 200%. The reason for this X_{δ_T} assumption is as follows. The weight of this jet transport is about 400,000 lb, while thrust available and thrust required in approach flight are about 120,000 and 36,000 lb, respectively. Therefore, the thrust variation is more critical when decreasing speed and the available deceleration is on the order of 0.1 g, at most. The lag time constant T_L of Eq. (4) is assumed to be 8 s.

Scale length and mean square gust velocities are determined according to Ref. 10. When the clear air turbulence model of the Dryden form is applied at an altitude of 1000 ft, the following are obtained:

$$L_w=1000\text{ ft} \qquad L_u=1450\text{ ft} \qquad \sigma_u^2/L_u=\sigma_w^2/L_w \qquad (35)$$

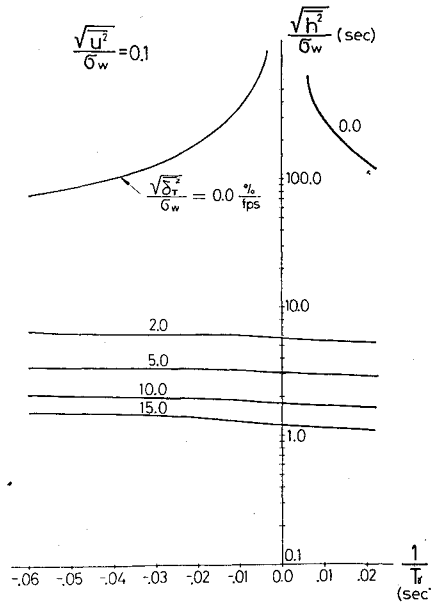

Fig. 6 Rms height error vs $1/T_\gamma$ (tight velocity control).

All the results shown in this paper are normalized by σ_w^2 or σ_w.

In Secs. A through C which follow, it is assumed $u_g=0$ and the results include the effect of w_g only. In Secs. D and E, both u_g and w_g are included in the calculations.

A. Effect of Throttle Loop Closure

As a first step, the term $\overline{u^2}/u_{ac}^2$ is eliminated from Eq. (34), and the relation between $\overline{h^2}/(\sigma_w^2 L_w/U)$ and $1/T_\gamma$ for constant $\delta_{T_{ac}}$ was calculated for a given δe_{ac}. The results are shown in Fig. 5. δe_{ac} is fixed at 16 deg. It was ascertained separately that this δe_{ac} was almost equivalent to set $\rho \cong 12.8$ rad^{-2} in Eq. (16) or $\overline{\delta_e^2}/(\sigma_w^2 L_w/U)\cong 3\times 10^{-2}$ s/ft^2 instead of Eq. (17); that is, more control is assumed than the criterion of Eq. (17). Therefore, $\delta e_{ac}=16$ deg is able to attain the performance close to that indicated by Eq. (31). Unless noted, $\delta e_{ac}=16$ deg is assumed throughout this paper, although the influence of δe_{ac} will be discussed later.

In the upper left portion of Fig. 5, some numerical values of $\sqrt{\overline{u^2}}/\sigma_w$ and $\sqrt{\overline{\delta_T^2}}/\sigma_w$ are shown with $\delta_{T_{ac}}$ for the case when $1/T_\gamma=-0.0118$ s^{-1}. It should be noted that the performance in mean square height error is drastically improved by the introduction of throttle control. However, it should also be noted that these calculations do not reflect the effect of velocity control, since $\overline{u^2}/u_{ac}^2$ have been eliminated from J [Eq. (34)].

A numerical check concerning the ordinate of Fig. 5 is in order. Typical numerical values are $\sqrt{\overline{h^2}}=30$ ft, $\sigma_w=10$ fps, $L_w=1000$ ft, and $U=200$ fps. This leads to $\overline{h^2}/(\sigma_w^2 L_w/U)\cong 2$ s. Thus, the region near the origin of the ordinate has the practical importance. For this reason, logarithmic scales are used in the figures which follow.

B. Effect of Velocity Control

Since the main role of the throttle is velocity control in the flight condition under study, $\overline{u^2}/u_{ac}^2$ is hereafter contained in the criterion J. δe_{ac} is fixed to 16 deg, as mentioned previously.

For a fixed value of Z_u (or $1/T_\gamma$), regulator analysis gives $\overline{u^2}$, $\overline{\delta_T^2}$, and $\overline{h^2}$ if u_{ac} and $\delta_{T_{ac}}$ are specified. Using this routine iteratively, a computer program was written which realized the particular combinations of $\overline{u^2}$ and $\overline{\delta_T^2}$ (by changing u_{ac} and $\delta_{T_{ac}}$). Figures 6-8 are the results of these calculations and show the relation between $\sqrt{\overline{h^2}}/\sigma_w$ and $1/T_\gamma$ when $\sqrt{\overline{\delta_T^2}}/\sigma_w$ and $\sqrt{\overline{u^2}}/\sigma_w$ are specified.

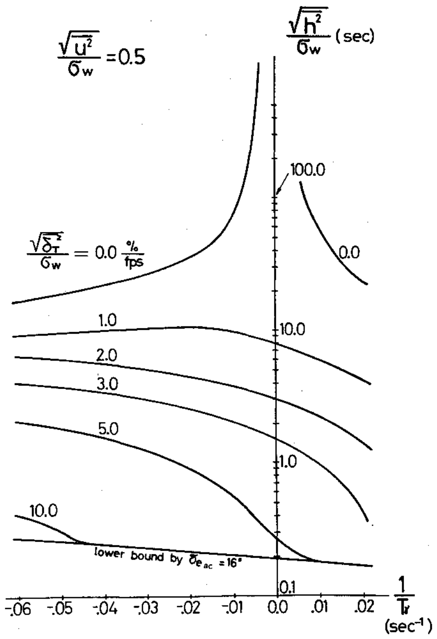

Fig. 7 Rms height error vs $1/T_\gamma$ (less tight velocity control).

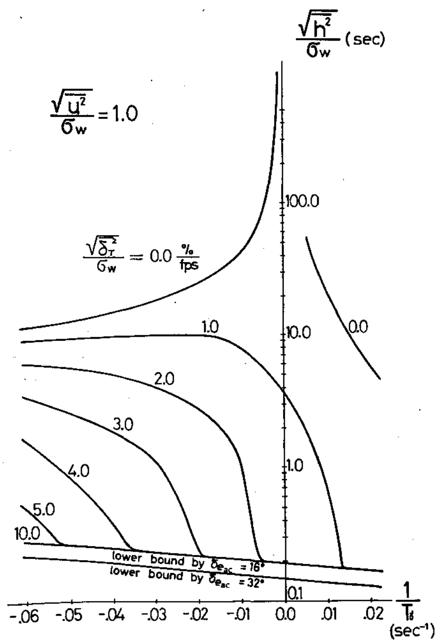

Fig. 8 Rms height error vs $1/T_\gamma$ (loose velocity control).

Figure 6 is the result for the case where the velocity is controlled tightly ($\sqrt{\overline{u^2}}/\sigma_w = 0.1$). Root mean square (rms) height error is not influenced much by the backside parameter $1/T_\gamma$, although the increase in rms throttle δ_T reduces rms height to some extent.

Figure 7 is the case where the velocity is controlled less tightly ($\sqrt{\overline{u^2}}/\sigma_w = 0.5$). Root mean square height error is reduced with the increase in $1/T_\gamma$, as well as with the increase in throttle.

Figure 8 is the case where the velocity is loosely controlled ($\sqrt{\overline{u^2}}/\sigma_w = 1.0$). As can be seen, $1/T_\gamma$ has a strong effect upon rms height error.

In the calculations of Figs. 7 and 8, the minimum realizable height error is almost determined by the value of δe_{ac} used; that is, $\sqrt{\overline{h^2}}/\sigma_w$ cannot be reduced beyond a boundary even if $\sqrt{\overline{\delta_T^2}}/\sigma_w$ is increased (by the reduction in $\delta_{T_{ac}}$). Such a boundary is noted in Figs. 7 and 8 as "lower bound by $\delta e_{ac} = 16$ deg." In Fig. 8, the boundary corresponding to $\delta e_{ac} = 32$ deg is also shown.

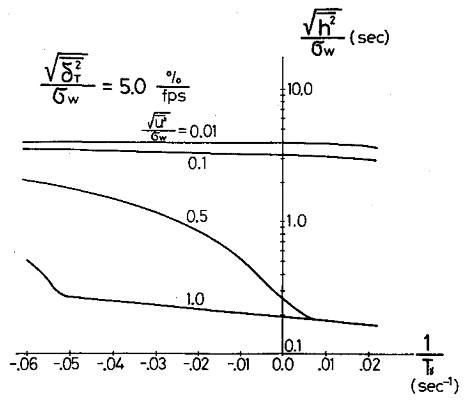

Fig. 9 Effect of velocity control on rms height error.

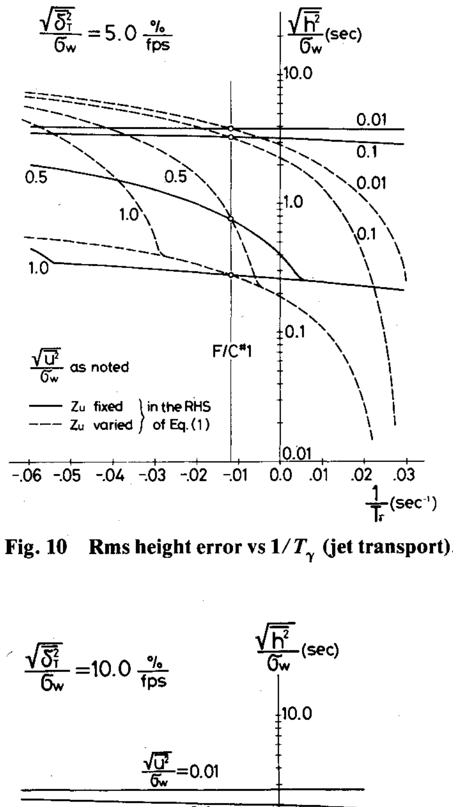

Fig. 10 Rms height error vs $1/T_\gamma$ (jet transport).

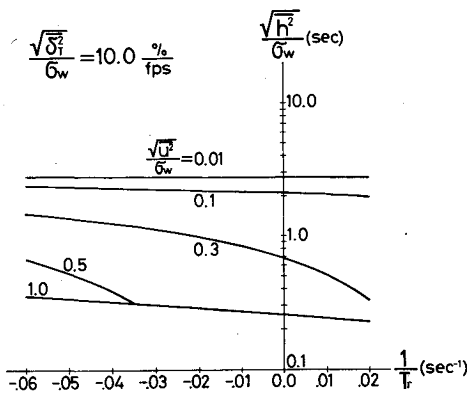

Fig. 11 Rms height error vs $1/T_\gamma$ (jet transport).

C. Height Error vs $1/T_\gamma$ When rms Throttle is Fixed

Figure 9 shows the $\sqrt{\overline{h^2}}/\sigma_w$ vs $1/T_\gamma$ trend as a function of $\sqrt{\overline{u^2}}/\sigma_w$, when $\sqrt{\overline{\delta_T^2}}/\sigma_w$ is fixed to 5%/fps. These curves were cross-plotted from Figs. 6-8, except for the case of $\sqrt{\overline{u^2}}/\sigma_w = 0.01$. This figure denotes typical relations between rms height, rms velocity, and $1/T_\gamma$ when rms throttle is kept constant. As an example application of Fig. 9, let us suppose

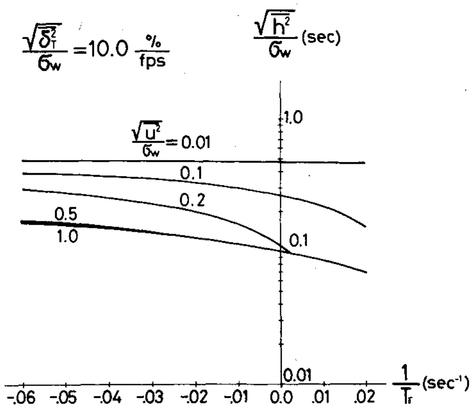

Fig. 12 Rms height error vs $1/T_\gamma$ (fighter).

an airplane whose backside parameter $1/T_\gamma$ and available rms throttle $\sqrt{\overline{\delta_T^2}}$ are -0.02 s^{-1} and 50%, respectively. When the rms gust σ_w is 10 fps and the velocity error $\sqrt{\overline{u^2}}$ is to be controlled within 1 fps, rms height error will be more than 30 ft. In order to keep height error within 10 ft, rms velocity error must be relaxed so that $\sqrt{\overline{u^2}} > 5$ fps.

Figure 9 suggests that it is difficult to tightly control height and velocity disturbances for large jet transport. This is mainly because the available thrust change is limited in these airplanes.

D. Effect of Horizontal Gust

Hereafter, the effect of horizontal gust u_g is included. Figure 10 is a result of calculations similar to Fig. 9, but including both u_g and w_g. The $\sqrt{\overline{h^2}}/\sigma_w$ vs $1/T_\gamma$ trend is essentially unchanged by the inclusion of horizontal gust. As noted previously, the curves (solid lines) are obtained with Z_u constant in the right-hand side (RHS) of Eq. (1). For the purpose of reference, results of calculations are also shown with Z_u varying (according to Fig. 4) in the RHS, which are the dashed lines in the same figure.

Figure 11 shows similar results when $\sqrt{\overline{\delta_T^2}}/\sigma_w$ is increased to 10%/fps. Height control performance is slightly improved with the increase in rms throttle motion.

E. Result of Calculations for a Fighter

Figure 12 is obtained for a hypothetical fighter. The effect of both u_g and w_g are included. The thrust derivative X_{δ_T} is chosen so that $X_{\delta_T} = \pm 0.25\ g/\pm 100\%\ \delta_T$ and $T_L = 2.0$ s. The relations between Z_u and $1/T_\gamma$ are shown in Fig. 4. Except as previously noted, the stability derivatives agree with those of an F4C, F/C #1, as shown in Ref. 6. Height control performance is better in Fig. 12 when compared with Fig. 11. This is probably because the fighter assumed larger X_{δ_T} and smaller T_L than the jet transport did.

Concluding Remarks

When flying through turbulence while controlling altitude by means of elevator alone, Eqs. (31) or (32) hold approximately, without regard to airplane type or flight condition in landing approach situations. The relation between mean square height error and the backside parameter should be attributed to the fact that $1/T_\gamma$, when negative, is a nonminimum phase zero of the system open-loop transfer function. Furthermore, when $1/T_\gamma$ is zero, the system is "uncontrollable" in the terminology of modern control theory.

Height control by elevator alone is, therefore, most critical when $1/T_\gamma$ is negative and close to zero. In these cases, throttle control has to be included. In these situations, it is generally difficult to tightly control airplane velocity and height simultaneously, and either one or the other usually must be sacrificed. To realize tight control in height, the velocity control requirement has to be relaxed and the backside parameter has to be properly large. This is mainly because the thrust change available is very limited in usual airplanes.

It should be noted that there is a distinct difference between the flight-path angle control and height control. In the path angle control, it may be possible to specify the performance in terms of the backside parameter only, as is done in Ref. 10. In height control, however, this seems unreasonable. It is the authors' opinion that height control characteristics must be specified, at least in terms of the available acceleration due to thrust change, the accuracy of the velocity control, and the backside parameter.

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